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## An M/G/1 Queue with Server Breakdown and Multiple Working Vacation

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### Abstract

This paper deals with the steady state behavior of an  $M/G/1$  multiple working vacation queue with server breakdown. The server works with different service times rather than completely stopping service during a vacation. Both service times in a vacation period and in a regular service period are assumed to be generally distributed random variables. The system may breakdown at random and repair time is arbitrary. Further, just after completion of a customer's service the server may take a multiple working vacation. Supplementary variable technique is employed to find the probability generating function for the number of customers in the system. The mean number of customers in the system is calculated. Some particular cases of interest are discussed. Numerical results are also presented.

**Keywords:** Poisson arrivals, Random breakdown, Repair time, Working Vacation, Supplementary Variable Technique

**MSC 2010 No.:** 60K25, 60K30

### 1. Introduction

In most of the queueing literature it is assumed that the server is available in the service station on a permanent basis and service station never fails. However, these assumptions are unrealistic. In practical situations we often meet the case where service stations may fail or slow down,

during the time, at which the repairing works are carried out. Such phenomenon always occur in the areas of computer communication networks and flexible manufacturing systems. Vacation queueing models subject to breakdowns have been studied by many authors including Gaver (1959), Levy and Yechilai (1976), Fuhrman (1981), Doshi (1986), Shanthikumar (1988), Kramer (1989), Madan (1999) and Madan and Saleh (2001) to mention a few. Sengupta (1990), Takine and Sengupta (1997), Li et al. (1997), Madan (2003), Choudhury and Tadj (2009), and Thangaraj and Vanitha (2010) studied M/G/1 queue with breakdowns and vacations.

Recently a class of semi-vacation policies called working vacation (WV) have been introduced. During this period the server works with a lower rate rather than completely stopping service. Servi and Finn (2002) studied an M/M/1 queue with multiple working vacation and obtained the probability generating function for the number of customers in the system and the waiting time distribution. Some other notable works are by Wu and Takagi (2006), Tian et al. (2008), Begum and Parveen (2011) and Santhi and Pazhani Bala Murugan (2013, 2014).

In this paper we study a non-Markovian queue with multiple working vacation and random breakdown. The organization of the paper is as follows. In Section 2, we describe the model. In Section 3, we obtain the steady state probability generating function. Particular cases are discussed in Section 4. Some performance measures are obtained in Section 5, and in Section 6 numerical study is presented.

## 2. The Model description

We assume the following to describe the queueing model under study. Customers arrive at the system one by one according to a Poisson stream with arrival rate  $\lambda(> 0)$ . The service discipline is FCFS. The service time follows a general distribution. Let  $S_b(x)$ ,  $s_b(x)$  and  $S_b^*(\theta)$  be the distribution function, the probability density function and the Laplace Stieltjes Transform (LST) of the service time  $S_b$ .

Whenever the system becomes empty at a service completion instant the working vacation of the server is begun. The duration of the vacation time is assumed to follow an exponential distribution with rate  $\eta$ . At a vacation completion instant, if there are customers in the system a new busy period will start. Otherwise, it stays in working vacation. This type of vacation is called multiple working vacation. During the working vacation, the server provides service with a different service time  $S_v$  which follows a general distribution with distribution function  $S_v(x)$ . Let  $s_v(x)$  and  $S_v^*(\theta)$  denote the corresponding probability density function, and Laplace Stieltjes Transform respectively.

The system may breakdown at random and it is assumed to occur according to a Poisson stream with mean breakdown rate  $\alpha_1(> 0)$  during the regular service period and  $\alpha_2(> 0)$  during the WV period, respectively. Further, we assume that once the system breaks down, the customer whose service is interrupted comes back to the head of the queue and the system enters a repair process immediately. The repair time is also assumed to follow a general distribution. Let the repair time distribution functions be  $S_{r_1}(x)$  and  $S_{r_2}(x)$  during the regular service period and WV

period, respectively. Let  $s_{r_1}(x)$ ,  $s_{r_2}(x)$ ,  $S_{r_1}^*(\theta)$ , and  $S_{r_2}^*(\theta)$  denoted the corresponding densities and LSTs respectively.

Various stochastic processes involved in the system are assumed to be independent of each other.

### 3. The System Size Distribution

The system size distribution at an arbitrary time can be obtained by using the supplementary variable technique, that is, from the joint distribution of the queue length and the remaining service time of the customer in service if the server is busy/working vacation. We define the following random variables.

$N(t)$  — the system size at time  $t$

$S_b^0(t)$  — the remaining service time in regular service period.

$S_v^0(t)$  — the remaining service time in working vacation period.

$S_{r_1}^0(t)$  — the remaining repair time in regular service period.

$S_{r_2}^0(t)$  — the remaining repair time in working vacation period.

$$Y(t) = \begin{cases} 0 & \text{if the server is idle at time } t, \\ 1 & \text{if the server is busy at time } t, \\ 2 & \text{if the server is busy on working vacation period at time } t, \\ 3 & \text{if the server is waiting for completion of repairing work during the} \\ & \text{busy period at time } t, \\ 4 & \text{if the server is waiting for completion of repairing work during the} \\ & \text{working vacation period at time } t. \end{cases}$$

Supplementary variables  $S_b^0(t)$ ,  $S_v^0(t)$ ,  $S_{r_1}^0(t)$  and  $S_{r_2}^0(t)$  are introduced in order to obtain bivariate Markov process  $\{(N(t), \partial(t)); t \geq 0\}$  where

$$\partial(t) = \begin{cases} S_b^0(t) & \text{if } Y(t) = 1, \\ S_v^0(t) & \text{if } Y(t) = 2, \\ S_{r_1}^0(t) & \text{if } Y(t) = 3, \\ S_{r_2}^0(t) & \text{if } Y(t) = 4. \end{cases}$$

We define the following limiting probabilities:

$$\begin{aligned} Q_0 &= \lim_{t \rightarrow \infty} Pr\{N(t) = 0, Y(t) = 0\}, \\ P_n(x) &= \lim_{t \rightarrow \infty} Pr\{N(t) = n, Y(t) = 1, x < S_b^0(t) \leq x + dx\}; \quad n \geq 1, \\ Q_n(x) &= \lim_{t \rightarrow \infty} Pr\{N(t) = n, Y(t) = 2, x < S_v^0(t) \leq x + dx\}; \quad n \geq 1, \\ R_{1,n}(x) &= \lim_{t \rightarrow \infty} Pr\{N(t) = n, Y(t) = 3, x < S_{r_1}^0(t) \leq x + dx\}; \quad n \geq 1, \\ \text{and } R_{2,n}(x) &= \lim_{t \rightarrow \infty} Pr\{N(t) = n, Y(t) = 4, x < S_{r_2}^0(t) \leq x + dx\}; \quad n \geq 1. \end{aligned}$$

Under the assumption that steady state conditions are reached, we have the following system of differential difference equations:

$$\lambda Q_0 = P_1(0) + Q_1(0), \quad (1)$$

$$-\frac{d}{dx}Q_1(x) = -(\lambda + \alpha_2 + \eta)Q_1(x) + Q_2(0)s_v(x) + \lambda Q_0 s_v(x) + R_{2,1}(0)s_v(x), \quad (2)$$

$$-\frac{d}{dx}Q_n(x) = -(\lambda + \alpha_2 + \eta)Q_n(x) + Q_{n+1}(0)s_v(x) + \lambda Q_{n-1}(x) + R_{2,n}(0)s_v(x) ; n \geq 2, \quad (3)$$

$$-\frac{d}{dx}P_1(x) = -(\lambda + \alpha_1)P_1(x) + P_2(0)s_b(x) + \eta s_b(x) \int_0^\infty Q_1(y)dy + R_{1,1}(0)s_b(x), \quad (4)$$

$$-\frac{d}{dx}P_n(x) = -(\lambda + \alpha_1)P_n(x) + P_{n+1}(0)s_b(x) + \eta s_b(x) \int_0^\infty Q_n(y)dy + \lambda P_{n-1}(x), \\ + R_{1,n}(0)s_b(x) ; n \geq 2, \quad (5)$$

$$-\frac{d}{dx}R_{2,1}(x) = -(\lambda + \eta)R_{2,1}(x) + \alpha_2 s_{r_2}(x) \int_0^\infty Q_1(x)dx, \quad (6)$$

$$-\frac{d}{dx}R_{2,n}(x) = -(\lambda + \eta)R_{2,n}(x) + \lambda R_{2,n-1}(x) + \alpha_2 s_{r_2}(x) \int_0^\infty Q_n(x)dx ; n \geq 2, \quad (7)$$

$$-\frac{d}{dx}R_{1,1}(x) = -\lambda R_{1,1}(x) + \alpha_1 s_{r_1}(x) \int_0^\infty P_1(x)dx + \eta s_{r_1}(x) \int_0^\infty R_{2,1}(y)dy, \quad (8)$$

$$-\frac{d}{dx}R_{1,n}(x) = -\lambda R_{1,n}(x) + \lambda R_{1,n-1}(x) + \alpha_1 s_{r_1}(x) \int_0^\infty P_n(x)dx \\ + \eta s_{r_1}(x) \int_0^\infty R_{2,n}(y)dy ; n \geq 2. \quad (9)$$

We define the Laplace Stieltjes transforms and the probability generating functions as follows. For  $i = 1, 2$ ,

$$S_b^*(\theta) = \int_0^\infty e^{-\theta x} s_b(x)dx ; \quad S_v^*(\theta) = \int_0^\infty e^{-\theta x} s_v(x)dx ; \quad S_{r_i}^*(\theta) = \int_0^\infty e^{-\theta x} s_{r_i}(x)dx ; \\ Q_n^*(\theta) = \int_0^\infty e^{-\theta x} Q_n(x)dx ; \quad Q_n^*(0) = \int_0^\infty Q_n(x)dx , \quad P_n^*(\theta) = \int_0^\infty e^{-\theta x} P_n(x)dx ; \\ P_n^*(0) = \int_0^\infty P_n(x)dx ; \quad R_{i,n}^*(\theta) = \int_0^\infty e^{-\theta x} R_{i,n}(x)dx ; \quad R_{i,n}^*(0) = \int_0^\infty R_{i,n}(x)dx ;$$

$$\begin{aligned}
Q^*(z, \theta) &= \sum_{n=1}^{\infty} Q_n^*(\theta) z^n, & Q(z, 0) &= \sum_{n=1}^{\infty} Q_n(0) z^n & Q^*(z, 0) &= \sum_{n=1}^{\infty} Q_n^*(0) z^n \\
P^*(z, \theta) &= \sum_{n=1}^{\infty} P_n^*(\theta) z^n & P(z, 0) &= \sum_{n=1}^{\infty} P_n(0) z^n, & P^*(z, 0) &= \sum_{n=1}^{\infty} P_n^*(0) z^n ; \\
R_i^*(z, \theta) &= \sum_{n=1}^{\infty} R_{i,n}^*(\theta) z^n, & R_i(z, 0) &= \sum_{n=1}^{\infty} R_{i,n}(0) z^n, & R_i^*(z, 0) &= \sum_{n=1}^{\infty} R_{i,n}^*(0) z^n.
\end{aligned}$$

Taking LST of (2) to (9), we get

$$\theta Q_1^*(\theta) - Q_1(0) = (\lambda + \alpha_2 + \eta) Q_1^*(\theta) - Q_2(0) S_v^*(\theta) - \lambda Q_0 S_v^*(\theta) - R_{2,1}(0) S_v^*(\theta), \quad (10)$$

$$\begin{aligned}
\theta Q_n^*(\theta) - Q_n(0) &= (\lambda + \alpha_2 + \eta) Q_n^*(\theta) \\
&\quad - Q_{n+1}(0) S_v^*(\theta) - \lambda Q_{n-1}^*(\theta) - R_{2,n}(0) S_v^*(\theta); \quad n \geq 2,
\end{aligned} \quad (11)$$

$$\theta P_1^*(\theta) - P_1(0) = (\lambda + \alpha_1) P_1^*(\theta) - P_2(0) S_b^*(\theta) - \eta S_b^*(\theta) Q_1^*(0) - R_{1,1}(0) S_b^*(\theta), \quad (12)$$

$$\begin{aligned}
\theta P_n^*(\theta) - P_n(0) &= (\lambda + \alpha_1) P_n^*(\theta) - P_{n+1}(0) S_b^*(\theta) - \eta S_b^*(\theta) Q_n^*(0) \\
&\quad - \lambda P_{n-1}^*(\theta) - R_{1,n}(0) S_b^*(\theta); \quad n \geq 2,
\end{aligned} \quad (13)$$

$$\theta R_{2,1}^*(\theta) - R_{2,1}(0) = (\lambda + \eta) R_{2,1}^*(\theta) - \alpha_2 S_{r_2}^*(\theta) Q_1^*(0), \quad (14)$$

$$\theta R_{2,n}^*(\theta) - R_{2,n}(0) = (\lambda + \eta) R_{2,n}^*(\theta) - \lambda R_{2,n-1}^*(\theta) - \alpha_2 S_{r_2}^*(\theta) Q_n^*(0); \quad n \geq 2, \quad (15)$$

$$\theta R_{1,1}^*(\theta) - R_{1,1}(0) = \lambda R_{1,1}^*(\theta) - \alpha_1 S_{r_1}^*(\theta) P_1^*(0) - \eta S_{r_1}^*(\theta) R_{2,1}^*(0), \quad (16)$$

$$\begin{aligned}
\theta R_{1,n}^*(\theta) - R_{1,n}(0) &= \lambda R_{1,n}^*(\theta) - \lambda R_{1,n-1}^*(\theta) \\
&\quad - \alpha_1 S_{r_1}^*(\theta) P_n^*(0) - \eta S_{r_1}^*(\theta) R_{2,n}^*(0); \quad n \geq 2.
\end{aligned} \quad (17)$$

$z^n$  times (11) summed over  $n$  from 2 to  $\infty$  and added up with  $z$  times (10) yields

$$\begin{aligned}
[\theta - (\lambda - \lambda z + \alpha_2 + \eta)] Q^*(z, \theta) \\
= \left[ \frac{z - S_v^*(\theta)}{z} \right] Q(z, 0) - S_v^*(\theta) [\lambda z Q_0 + R_2(z, 0) - Q_1(0)].
\end{aligned} \quad (18)$$

Inserting  $\theta = (\lambda - \lambda z + \alpha_2 + \eta) = (a(z) + \alpha_2)$  in (18), we get

$$Q(z, 0) = \frac{z S_v^*(a(z) + \alpha_2) [\lambda z Q_0 + R_2(z, 0) - Q_1(0)]}{z - S_v^*(a(z) + \alpha_2)}. \quad (19)$$

$z^n$  times (15) summed over  $n$  from 2 to  $\infty$ , added up with  $z$  times (14), gives

$$[\theta - (\lambda - \lambda z + \eta)] R_2^*(z, \theta) = R_2(z, 0) - \alpha_2 S_{r_2}^*(\theta) Q^*(z, 0). \quad (20)$$

Inserting  $\theta = (\lambda - \lambda z + \eta) = a(z)$  in (20), we get

$$R_2(z, 0) = \alpha_2 S_{r_2}^*(a(z)) Q^*(z, 0) \quad (21)$$

Substituting (21) in (20) and putting  $\theta = 0$ , we get

$$R_2^*(z, 0) = \frac{\alpha_2 Q^*(z, 0) (1 - S_{r_2}^*(a(z)))}{(a(z))}. \quad (22)$$

Substituting (21) in (19), we get

$$Q(z, 0) = \frac{zS_v^*(a(z) + \alpha_2)[\lambda zQ_0 + \alpha_2 S_{r_2}^*(a(z))Q^*(z, 0) - Q_1(0)]}{z - S_v^*(a(z) + \alpha_2)}. \quad (23)$$

Substituting (21) and (23) in (18), we get

$$\begin{aligned} & [\theta - (a(z) + \alpha_2)]Q^*(z, \theta) \\ &= \frac{[z(S_v^*(a(z) + \alpha_2) - S_v^*(\theta))][\alpha_2 S_{r_2}^*(a(z))Q^*(z, 0) + \lambda zQ_0 - Q_1(0)]}{z - S_v^*(a(z) + \alpha_2)}. \end{aligned}$$

Putting  $\theta = 0$ , we get

$$\begin{aligned} & Q^*(z, 0) \\ &= \frac{z(1 - S_v^*(a(z) + \alpha_2))(\lambda zQ_0 - Q_1(0))}{[(a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) - \alpha_2 z(1 - S_v^*(a(z) + \alpha_2))S_{r_2}^*(\lambda - \lambda z + \eta)]}. \end{aligned} \quad (24)$$

The denominator of the above equation has a unique root  $z_1$  in  $(0, 1)$  and thus  $Q_1(0) = \lambda z_1 Q_0$ .

Substituting this in (24), we get

$$\begin{aligned} & Q^*(z, 0) \\ &= \frac{\lambda z(z - z_1)(1 - S_v^*(a(z) + \alpha_2))Q_0}{[(a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) - \alpha_2 z(1 - S_v^*(a(z) + \alpha_2))S_{r_2}^*(\lambda - \lambda z + \eta)]}. \end{aligned} \quad (25)$$

Substituting (25) in (22), we get

$$R_2^*(z, 0) = \frac{Q_0 \alpha_2 (1 - S_{r_2}^*(a(z))) \lambda z(z - z_1)(1 - S_v^*(a(z) + \alpha_2))}{a(z) \left[ (a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) - \alpha_2 z(1 - S_v^*(a(z) + \alpha_2))S_{r_2}^*(a(z)) \right]}. \quad (26)$$

$z^n$  times (13) summed over  $n$  from 2 to  $\infty$ , is added up with  $z$  times (12) yields

$$\begin{aligned} & [\theta - (\lambda - \lambda z + \alpha_1)]P^*(z, \theta) \\ &= \left[ \frac{z - S_b^*(\theta)}{z} \right] P(z, 0) - S_b^*(\theta) [\eta Q^*(z, 0) + R_1(z, 0) - P_1(0)]. \end{aligned} \quad (27)$$

Inserting  $\theta = (\lambda - \lambda z + \alpha_1) = a_1(z)$  and substituting  $\lambda(1 - z_1)Q_0 = P_1(0)$  in (27), we get

$$P(z, 0) = \frac{zS_b^*(a_1(z))[\eta Q^*(z, 0) + R_1(z, 0) - \lambda(1 - z_1)Q_0]}{z - S_b^*(a_1(z))}. \quad (28)$$

$z^n$  times (17) summed over  $n$  from 2 to  $\infty$ , added with  $z$  times (16), results in

$$[\theta - (\lambda - \lambda z)]R_1^*(z, \theta) = R_1(z, 0) - \alpha_1 S_{r_1}^*(\theta)P^*(z, 0) - \eta S_{r_1}^*(\theta)R_2^*(z, 0). \quad (29)$$

Inserting  $\theta = (\lambda - \lambda z)$  in (29), we get

$$R_1(z, 0) = S_{r_1}^*(\lambda - \lambda z) [\alpha_1 P^*(z, 0) + \eta R_2^*(z, 0)]. \quad (30)$$

Substituting (30) in (29) and putting  $\theta = 0$  in (29), we get

$$R_1^*(z, 0) = \frac{(1 - S_{r_1}^*(\lambda - \lambda z))[\alpha_1 P^*(z, 0) + \eta R_2^*(z, 0)]}{(\lambda - \lambda z)}. \quad (31)$$

Substituting (22), (25), (28), (30) and  $\lambda(1 - z_1)Q_0 = P_1(0)$  and putting  $\theta = 0$  in (27), we get

$$P^*(z, 0) = \frac{Nr(z)}{Dr(z)}, \quad (32)$$

where

$$\begin{aligned} Nr(z) &= Q_0 \lambda z (1 - S_b^*(a_1(z))) \{ \eta z (z - z_1) (1 - S_v^*(a(z) + \alpha_2)) [a(z) + \alpha_2 S_{r_1}^*(\lambda - \lambda z) \\ &\quad \times (1 - S_{r_2}^*(a(z)))] - (a(z))(1 - z_1) [(a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) \\ &\quad - \alpha_2 z (1 - S_v^*(a(z) + \alpha_2)) S_{r_2}^*(a_1(z))] \}, \\ Dr(z) &= a(z) \{ (a_1(z))(z - S_b^*(a_1(z))) - z \alpha_1 (1 - S_b^*(a_1(z))) S_{r_1}^*(\lambda - \lambda z) \} \\ &\quad \times \{ (a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) - \alpha_2 z (1 - S_v^*(a(z) + \alpha_2)) S_{r_2}^*(a(z)) \}. \end{aligned}$$

Substituting (26) and (32) in (31), we get

$$\begin{aligned} R_1^*(z, 0) &= \frac{Q_0(1 - S_{r_1}^*(\lambda - \lambda z))}{D_1(z)D_2(z)D_3(z)} \left\{ \alpha_1 \lambda z (1 - S_b^*(a_1(z))) \{ \eta z (z - z_1) (1 - S_v^*(a(z) + \alpha_2)) \right. \\ &\quad \times [(a(z) + \alpha_2 S_{r_1}^*(\lambda - \lambda z))(1 - S_{r_2}^*(a(z)))] - (a(z)) \times (1 - z_1) [(a(z) + \alpha_2) \\ &\quad \times (z - S_v^*(a(z) + \alpha_2)) + \alpha_2 \eta \lambda z (z - z_1) (1 - S_v^*(a(z) + \alpha_2)) \times (1 - S_{r_2}^*(a(z)))] \\ &\quad \left. \times \{ (a_1(z)) \times (z - S_b^*(a_1(z))) - \alpha_1 z (1 - S_b^*(a_1(z))) \times S_{r_1}^*(\lambda - \lambda z) \} \right\}, \quad (33) \end{aligned}$$

where

$$D_1(z) = a(z) \{ a_1(z)(z - S_b^*(a_1(z))) - z \alpha_1 (1 - S_b^*(a_1(z))) S_{r_1}^*(\lambda - \lambda z) \}, \quad (34)$$

$$D_2(z) = \{ (a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) - \alpha_2 z (1 - S_v^*(a(z) + \alpha_2)) S_{r_2}^*(a(z)) \}, \quad (35)$$

$$D_3(z) = (\lambda - \lambda z). \quad (36)$$

We define

$$P_B(z) = P^*(z, 0) + R_1^*(z, 0) \quad (37)$$

as the probability generating function for the number of customers in the system when the server is in regular service period,

$$P_V(z) = Q^*(z, 0) + R_2^*(z, 0) + Q_0 \quad (38)$$

as the probability generating function for the number of customers in the system when the server is on working vacation period, and

$$P(z) = P_B(z) + P_V(z) \quad (39)$$

as the probability generating function for the number of customers in the system. We now use the normalizing condition  $P(1) = 1$  to determine the only unknown,  $Q_0$ , which appears in (39). Substituting  $z = 1$  in (39) and using L'Hôpital's rule, we obtain

$$Q_0 = \frac{1 - \rho_b}{\left[ \frac{\eta + \lambda(1 - z_1)}{\eta} \right] - \left[ \frac{C_1}{C_2} \right]}, \quad (40)$$



where

$$\begin{aligned} C_1 &= \lambda(1 - z_1) \{ (1 - S_b^*(\alpha_1)) S_v^*(\eta + \alpha_2) \times [\alpha_2 + \eta(1 + \alpha_1 E(S_{r_1}))] + \alpha_2 \alpha_1 E(S_{r_1}) \\ &\quad \times [S_v^*(\eta + \alpha_2) - S_b^*(\alpha_1) [1 - S_{r_2}^*(\eta)(1 - S_v^*(\eta + \alpha_2))] \} \}, \\ C_2 &= \alpha_1 S_b^*(\alpha_1) (1 - S_v^*(\eta + \alpha_2)) (\eta + \alpha_2 (1 - S_{r_2}^*(\eta))), \end{aligned}$$

and  $\rho_b = \frac{\lambda(1 - S_b^*(\alpha_1))(1 + \alpha_1 E(S_{r_1}))}{\alpha_1 S_b^*(\alpha_1)}$ ,  $E(S_{r_1})$  is the mean repair time in regular service period. From (40) we obtain the system stability condition  $\rho_b < 1$ .

#### 4. Particular Cases

**Case (i):** If the system suffers no breakdowns, then letting  $\alpha_1 = 0$  and  $\alpha_2 = 0$  in (39), we have

$$P(z) = \frac{Q_0 \lambda z (1 - S_b^*(\lambda - \lambda z)) \times N r_1(z)}{(\lambda - \lambda z)(a(z))(z - S_b^*(\lambda - \lambda z))(z - S_v^*(a(z)))}, \quad (41)$$

where

$$\begin{aligned} N r_1(z) &= \left\{ \eta z (z - z_1) (1 - S_v^*(a(z))) - (1 - z_1) (a(z)) (z - S_v^*(a(z))) \right\} + (\lambda - \lambda z), \\ &\quad \times (z - S_b^*(\lambda - \lambda z)) \left\{ \lambda z (z - z_1) (1 - S_v^*(a(z))) + (a(z)) (z - S_v^*(a(z))) \right\}, \end{aligned}$$

$$\text{where } Q_0 = \frac{1 - \rho_b}{\left[ \frac{\lambda - \lambda z_1 + \eta}{\eta} - \frac{\rho_b (1 - z_1) S_v^*(\eta)}{1 - S_v^*(\eta)} \right]}, \quad \rho_b = \lambda E(S_b).$$

Equation (41) is a well-known probability generating function of the steady state system length distribution of an  $M/G/1$  queue with multiple working vacation (Takagi (2006)) irrespective of the notations.

**Case (ii):** If the server never does the work during vacation period then setting  $S_v^*(\lambda - \lambda z + \eta + \alpha_2) = 0$ ,  $\alpha_2 = 0$  and  $S_{r_2}^*(\lambda - \lambda z + \eta) = 0$  in (39) and by taking the repair time to be exponentially distributed, we get

$$P(z) = P_V(z) + P_B(z), \quad (42)$$

where

$$\begin{aligned} P_V(z) &= \frac{Q_0 (\lambda (1 - z_1) + \eta)}{\lambda - \lambda z + \eta} \\ P_B(z) &= \frac{Q_0 z [S_b^*(a_1(z)) - 1] [\lambda (1 - z_1) + \eta] [(\lambda - \lambda z)(\beta + \lambda - \lambda z) + \alpha_1 (\lambda - \lambda z)]}{\left\{ (a(z)) [(\lambda - \lambda z)(z - S_b^*(a_1(z))) (\beta + \lambda - \lambda z) \right.} \\ &\quad \left. + \alpha_1 z (\lambda - \lambda z) - \alpha_1 S_b^*(a_1(z)) (\beta (1 - z) + \lambda - \lambda z) \right\}} \\ Q_0 &= \frac{1 - \rho_b}{\left[ \frac{\eta + \lambda (1 - z_1)}{\eta} \right]}, \quad \rho_b = \frac{\lambda (1 - S_b^*(\alpha_1)) (\alpha_1 + \beta)}{\alpha_1 \beta S_b^*(\alpha_1)}. \end{aligned}$$

Equation (42) is a well-known probability generating function of the steady state system length distribution of an  $M/G/1$  queue with Server Vacation and Random Breakdown (Thangaraj (2010) no second stage service) irrespective of the notations.

**Case (iii):** If the system suffers no breakdowns and the server never takes a vacation then on setting  $\alpha_1 = 0, \alpha_2 = 0$  and taking limit  $\eta \rightarrow \infty$  in (39) we get

$$P(z) = \frac{(1 - \lambda E(S_b))(1 - z)S_b^*(\lambda - \lambda z)}{S_b^*(\lambda - \lambda z) - z}. \quad (43)$$

Equation (43) is a well-known probability generating function of the steady state system length distribution of an  $M/G/1$  queue (Medhi (1982)) irrespective of the notations where  $E(S_{r_1})$  is the mean repair time in regular service period.

## 5. Performance Measures

Let  $L_v$  and  $L_b$  denote the mean system size during the working vacation and regular service period respectively and let  $W_v$  and  $W_b$  be the mean waiting time of the customers in the system during working vacation period and regular service period respectively.

$$L_v = \frac{d}{dz}[P_V(z)]|_{z=1} = \frac{d}{dz} \left[ \frac{A(z)}{D_2(z)} + \frac{B(z)}{a(z)D_2(z)} \right] Q_0|_{z=1},$$

where

$$\begin{aligned} A(z) &= \lambda z(z - z_1)(1 - S_v^*(a(z) + \alpha_2)), \\ B(z) &= \alpha_2 \lambda z(z - z_1)(1 - S_{r_2}^*(a(z)))(1 - S_v^*(a(z) + \alpha_2)). \end{aligned}$$

$D_2(z)$  is given in (35). Therefore

$$L_v = Q_0 \left[ \frac{D_2(1)A'(1) - A(1)D_2'(1)}{(D_2(1))^2} + \frac{\eta(D_2(1)B'(1) - B(1)D_2'(1)) + \lambda B(1)D_2(1)}{(\eta D_2(1))^2} \right],$$

and applying Little's formula  $W_v = \frac{L_v}{\lambda}$ , we have

$$\begin{aligned} A(1) &= \lambda(1 - z_1)(1 - S_v^*(\eta + \alpha_2)), \\ A'(1) &= (1 - S_v^*(\eta + \alpha_2))(\lambda + \lambda(1 - z_1)) + \lambda^2(1 - z_1)S_v^{*'}(\eta + \alpha_2), \\ D_2(1) &= (1 - S_v^*(\eta + \alpha_2))[\eta + \alpha_2(1 - S_{r_2}^*(\eta))], \\ B(1) &= \alpha_2 \lambda(1 - z_1)(1 - S_{r_2}^*(\eta))(1 - S_v^*(\eta + \alpha_2)), \\ B'(1) &= \alpha_2 [\lambda(1 - z_1)[(1 - S_v^*(\eta + \alpha_2))(1 - S_{r_2}^*(\eta) + \lambda S_{r_2}^{*'}(\eta)) + \lambda(1 - S_{r_2}^*(\eta))S_v^{*'}(\eta + \alpha_2)] \\ &\quad + \lambda(1 - S_{r_2}^*(\eta))(1 - S_v^*(\eta + \alpha_2))], \\ L_b &= \frac{d}{dz}[P_B(z)]|_{z=1} = \frac{d}{dz} \left[ \frac{N_1(z)N_2(z)}{D_1(z)D_2(z)} + \frac{N_3(z)N_4(z)}{D_1(z)D_2(z)D_3(z)} \right] Q_0 \Big|_{z=1}, \end{aligned}$$

where

$$\begin{aligned}
 N_1(z) &= \lambda z(1 - S_b^*(a_1(z))), \\
 N_2(z) &= \eta z(z - z_1)(1 - S_v^*(a(z) + \alpha_2))[(a(z)) + \alpha_2 S_{r_1}^*(\lambda - \lambda z)(1 - S_{r_2}^*(a(z)))] - (a(z)) \\
 &\quad \times (1 - z_1)[(a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) - \alpha_2 z S_{r_2}^*(a(z))(1 - S_v^*(a(z) + \alpha_2))], \\
 N_3(z) &= (1 - S_{r_1}^*(\lambda - \lambda z)), \\
 N_4(z) &= \alpha_1 \lambda z(1 - S_b^*(a_1(z)))\{\eta z(z - z_1)(1 - S_v^*(a(z) + \alpha_2))[(a(z)) + \alpha_2 S_{r_1}^*(\lambda - \lambda z) \\
 &\quad \times (1 - S_{r_2}^*(a(z)))] - (a(z))(1 - z_1)((a(z) + \alpha_2)(z - S_v^*(a(z) + \alpha_2)) - \alpha_2 z \\
 &\quad \times (1 - S_v^*(a(z) + \alpha_2)) S_{r_2}^*(a(z))]\} + \alpha_2 \eta \lambda z(z - z_1)(1 - S_v^*(a(z) + \alpha_2)) \\
 &\quad \times (1 - S_{r_2}^*(a(z)))\{(\lambda - \lambda z + \alpha_1)(z - S_b^*(a_1(z))) - \alpha_1 z(1 - S_b^*(a_1(z))) S_{r_1}^*(\lambda - \lambda z)\},
 \end{aligned}$$

$D_1(z)$ ,  $D_2(z)$  and  $D_3(z)$  are given in equations (34), (35), and (36), respectively and

$$\begin{aligned}
 L_b &= Q_0 \frac{\left[ 2D_1'(1)N_2'(1)(D_2(1)N_1'(1) - N_1(1)D_2'(1)) + D_2(1)N_1(1)(D_1'(1)N_2''(1)) \right.}{4(D_1'(1)D_2(1))^2} \\
 &\quad \left. - N_2'(1)D_1''(1) \right] \\
 &\quad + Q_0 \frac{\left[ D_1'(1)D_2(1)D_3'(1)(N_3''(1)N_4'(1) + N_3'(1)N_4''(1)) - D_3'(1)N_3'(1)N_4'(1) \right.}{2(D_1'(1)D_2(1)D_3'(1))^2} \\
 &\quad \left. \times (D_1''(1)D_2(1) + 2D_1'(1)D_2'(1)) \right], \\
 W_b &= \frac{L_b}{\lambda}, \\
 N_1(1) &= \lambda(1 - S_b^*(\alpha_1)), \\
 N_1'(1) &= \lambda(1 - S_b^*(\alpha_1) + \lambda^2 S_b^{*'}(\alpha_1)), \\
 N_2'(1) &= (\eta + \alpha_2(1 - S_{r_2}^*(\eta)))(1 - S_v^*(\eta + \alpha_2))(\eta + \lambda(1 - z_1)) - \eta(1 - z_1) \\
 &\quad \times [(\eta + \alpha_2)S_v^*(\eta + \alpha_2) - \lambda \alpha_2 E(S_{r_1})(1 - S_v^*(\eta + \alpha_2))(1 - S_{r_2}^*(\eta))] \\
 N_2''(1) &= 2\eta(\eta + \alpha_2(1 - S_{r_2}^*(\eta)))(1 - S_v^*(\eta + \alpha_2) + \lambda(1 - z_1)S_v^{*'}(\eta + \alpha_2) + \lambda S_v^{*'}(\eta + \alpha_2)) \\
 &\quad + 2\eta(-\lambda + \lambda \alpha_2 E(S_{r_1})(1 - S_{r_2}^*(\eta)))[(1 - S_v^*(\eta + \alpha_2)) + (1 - z_1) + \lambda(1 - z_1) \\
 &\quad \times S_v^{*'}(\eta + \alpha_2)] + 2\eta\lambda(1 - S_v^*(\eta + \alpha_2))\alpha_2 S_{r_2}^{*'}(\eta) + \eta(1 - z_1)\lambda^2 S_v^{*'}(\eta + \alpha_2) \\
 &\quad \times \alpha_2 S_{r_2}^{*'}(\eta) + \lambda^2 \alpha_2 \eta(1 - z_1)(1 - S_v^*(\eta + \alpha_2))[E(S_{r_1})^2(1 - S_{r_2}^*(\eta)) + 2E(S_{r_1}) \\
 &\quad \times S_{r_2}^{*'}(\eta)] + 2\lambda(1 - z_1)[(1 - S_v^*(\eta + \alpha_2))(-\lambda - \alpha_2 S_{r_2}^*(\eta) + \lambda \alpha_2 S_{r_2}^{*'}(\eta)) \\
 &\quad + \alpha_2(1 + \lambda S_v^{*'}(\eta + \alpha_2)) - \lambda \alpha_2 S_v^{*'}(\eta + \alpha_2)S_{r_2}^*(\eta)] - \eta(1 - z_1)[\lambda^2 \alpha_2 S_v^{*''}(\eta + \alpha_2) \\
 &\quad \times S_{r_2}^*(\eta) - 2\lambda \alpha_2 S_v^{*'}(\eta + \alpha_2)S_{r_2}^*(\eta)], \\
 N_3'(1) &= -\lambda E(S_{r_1}), \\
 N_3''(1) &= -\lambda^2 E(S_{r_1})^2,
 \end{aligned}$$

$$N'_4(1) = \lambda\alpha_1(1 - S_b^*(\alpha_1))\{-\eta(1 - z_1)S_v^*(\eta + \alpha_2)(\eta + \alpha_2) + (1 - S_v^*(\eta + \alpha_2)) \\ \times (\eta + \alpha_2(1 - S_{r_2}^*(\eta)))(\eta + \lambda(1 - z_1))\} + \eta\alpha_2(1 - S_{r_2}^*(\eta))\lambda(1 - z_1) \\ \times (1 - S_v^*(\eta + \alpha_2))[\alpha_1 S_b^*(\alpha_1) - \lambda(1 - S_b^*(\alpha_1))],$$

$$N''_4(1) = 2(\lambda\alpha_1(1 - S_b^*(\alpha_1)) + \alpha_1\lambda^2 S_b^{*'}(\alpha_1))\{((1 - S_v^*(\eta + \alpha_2))(\eta + \eta(1 - z_1)) \\ + \eta(1 - z_1)\lambda S_v^{*'}(\eta + \alpha_2))(\eta + \alpha_2(1 - S_{r_2}^*(\eta))) + \eta(1 - z_1)(1 - S_v^*(\eta + \alpha_2)) \\ \times [-\lambda + \lambda\alpha_2 S_{r_2}^{*'}(\eta) + \lambda\alpha_2 E(S_{r_1})(1 - S_{r_2}^*(\eta))] + \lambda(1 - z_1)(1 - S_v^*(\eta + \alpha_2)) \\ \times \alpha_2(1 - S_{r_2}^*(\eta)) + \eta(1 - z_1)[\eta + \alpha_2 + \lambda S_v^{*'}(\eta + \alpha_2)(\eta + \alpha_2(1 - S_{r_2}^*(\eta)))] \\ - \alpha_2(1 - S_v^*(\eta + \alpha_2))(S_{r_2}^*(\eta) - \lambda S_{r_2}^{*'}(\eta))\} + \lambda\alpha_1(1 - S_b^*(\alpha_1)) \\ \times \{ (2\eta(1 - S_v^*(\eta + \alpha_2)) + 2\eta(1 - z_1)\lambda S_v^{*'}(\eta + \alpha_2) + 2\eta\lambda S_v^{*'}(\eta + \alpha_2) \\ - \eta(1 - z_1)\lambda^2 S_v^{*''}(\eta + \alpha_2))(\eta + \alpha_2(1 - S_{r_2}^*(\eta))) + 2((1 - S_v^*(\eta + \alpha_2)) \\ \times (\eta + \eta(1 - z_1)) + \eta\lambda(1 - z_1)S_v^{*'}(\eta + \alpha_2))(-\lambda + \lambda\alpha_2 E(S_{r_1})(1 - S_{r_2}^*(\eta)) \\ + \lambda\alpha_2 S_{r_2}^{*'}(\eta)) + \eta(1 - z_1)(1 - S_v^*(\eta + \alpha_2))[2\lambda^2\alpha_2 E(S_{r_1})S_{r_2}^{*'}(\eta) - \lambda^2\alpha_2 S_{r_2}^{*''}(\eta) \\ + \lambda^2\alpha_2 E(S_{r_1})^2(1 - S_{r_2}^*(\eta))] + 2\lambda(1 - z_1)[(1 - S_v^*(\eta + \alpha_2))[-\lambda - \alpha_2 S_{r_2}^*(\eta) \\ + \lambda\alpha_2 S_{r_2}^{*'}(\eta)] + \eta + \alpha_2 + \lambda S_v^{*'}(\eta + \alpha_2)(\eta + \alpha_2(1 - S_{r_2}^*(\eta)))] \} \\ + 2\{ \eta\lambda\alpha_2(1 - S_v^*(\eta + \alpha_2))((1 - S_{r_2}^*(\eta))(1 + (1 - z_1)) + \lambda S_{r_2}^{*'}(\eta)) \\ + \lambda^2(1 - z_1)\eta\alpha_2(1 - S_{r_2}^*(\eta))S_v^{*'}(\eta + \alpha_2) \} \\ \times \{ \lambda(1 - S_b^*(\alpha_1))(-1 - \alpha_1 E(S_{r_1})) + \alpha_1 S_b^*(\alpha_1) \} + \eta\alpha_2\lambda(1 - z_1)(1 - S_{r_2}^*(\eta)) \\ \times (1 - S_v^*(\eta + \alpha_2))\{-2\lambda(1 + \lambda S_b^{*'}(\alpha_1)) - 2\lambda\alpha_1 S_b^{*'}(\alpha_1) - 2\lambda\alpha_1(1 - S_b^*(\alpha_1))E(S_{r_1}) \\ - 2\lambda^2\alpha_1 S_b^{*'}(\alpha_1)E(S_{r_1}) - \lambda^2\alpha_1(1 - S_b^*(\alpha_1))E(S_{r_1})^2\},$$

$$D'_1(1) = \eta\{\alpha_1 - (1 - S_b^*(\alpha_1))(\lambda + \alpha_1(1 + \lambda E(S_{r_1})))\},$$

$$D''_1(1) = -2\lambda(\alpha_1 - (1 - S_b^*(\alpha_1))(\lambda + \alpha_1 + \lambda E(S_{r_1}))) \\ + \eta\{-2\lambda(1 + \lambda S_b^{*'}(\alpha_1)) - 2\lambda\alpha_1 S_b^{*'}(\alpha_1) - 2\lambda\alpha_1(1 - S_b^*(\alpha_1))E(S_{r_1}) \\ - 2\lambda^2\alpha_1 S_b^{*'}(\alpha_1)E(S_{r_1}) - \lambda^2\alpha_1(1 - S_b^*(\alpha_1))E(S_{r_1})^2\},$$

$$D_2(1) = (1 - S_v^*(\eta + \alpha_2))[\eta + \alpha_2(1 - S_{r_2}^*(\eta))],$$

$$D'_2(1) = (1 - S_v^*(\eta + \alpha_2))[-\lambda - \alpha_2 S_{r_2}^*(\eta) + \lambda\alpha_2 S_{r_2}^{*'}(\eta)] + \alpha_2 + \eta(1 + \lambda S_v^{*'}(\eta + \alpha_2)) \\ + \lambda\alpha_2 S_v^{*'}(\eta + \alpha_2)(1 - S_{r_2}^*(\eta)),$$

$$D'_3(1) = -\lambda.$$

## 6. Numerical Result

Assuming that the service time distribution for both regular service period and working vacation period as exponentially distributed and using the fact that

$$\begin{aligned} S_b^*(\alpha_1) &= \frac{\mu_b}{(\alpha_1 + \mu_b)}, & S_v^*(\eta + \alpha_2) &= \frac{\mu_v}{(\eta + \alpha_2 + \mu_v)}, & E(S_{r_1}) &= \frac{1}{\mu_{r_1}} \\ S_b^{*'}(\alpha_1) &= -\frac{\mu_b}{(\alpha_1 + \mu_b)^2}, & S_v^{*'}(\eta + \alpha_2) &= -\frac{\mu_v}{(\eta + \alpha_2 + \mu_v)^2}, & E(S_{r_1}^2) &= \frac{2}{\mu_{r_1}^2}, \\ S_{r_2}^*(\eta) &= \frac{\mu_{r_2}}{(\eta + \mu_{r_2})}, & S_v^{*''}(\eta + \alpha_2) &= \frac{2\mu_v}{(\eta + \alpha_2 + \mu_v)^3}, & S_{r_2}^{*'}(\eta) &= -\frac{\mu_{r_2}}{(\eta + \mu_{r_2})^2}, \end{aligned}$$

and by fixing the values of  $z_1 = 0.6$ ,  $\mu_v = 6$ ,  $\mu_b = 15$ ,  $\mu_{r_1} = 2$ ,  $\mu_{r_2} = 5$ ,  $\alpha_1 = 2$ ,  $\alpha_2 = 1$  and ranging the values of  $\lambda$  from 3.1 to 3.5 in steps of 0.1 and varying the values of  $\eta$  from 3.1 to 3.9 in steps of 0.2, we calculated the corresponding values of  $L_b$  and  $W_b$  for multiple working vacation and tabulated in Table 1 and in Table 2, respectively.

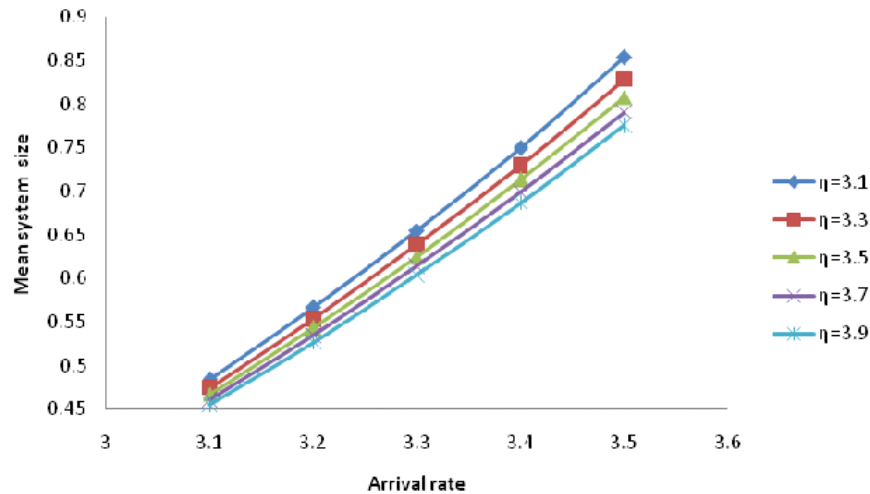
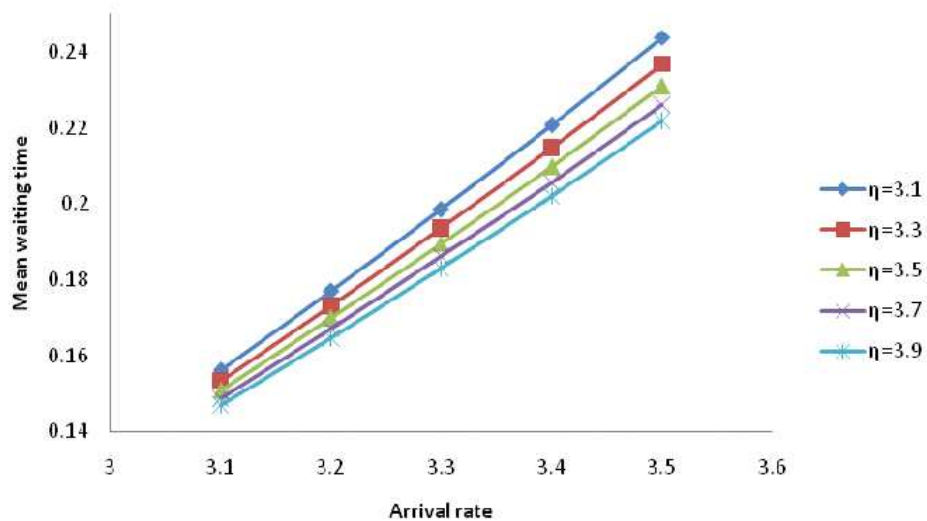
Table 1. Arrival rate ( $\lambda$ ) versus mean system size ( $L_b$ ) in regular service period

$\lambda \backslash \eta$	3.1	3.3	3.5	3.7	3.9
3.1	0.484350	0.475244	0.467617	0.461146	0.455593
3.2	0.566134	0.553773	0.543432	0.534666	0.527152
3.3	0.654603	0.638586	0.625199	0.613862	0.604151
3.4	0.750153	0.730055	0.713273	0.699071	0.686915
3.5	0.853213	0.828583	0.808033	0.790658	0.775796

Table 2. Arrival rate ( $\lambda$ ) versus mean waiting time ( $W_b$ ) in regular service period

$\lambda \backslash \eta$	3.1	3.3	3.5	3.7	3.9
3.1	0.156242	0.153304	0.150844	0.148757	0.146966
3.2	0.176917	0.173054	0.169823	0.167083	0.164735
3.3	0.198364	0.193511	0.189454	0.186019	0.183076
3.4	0.220633	0.214722	0.209786	0.205609	0.202034
3.5	0.243775	0.236738	0.230867	0.225902	0.221656

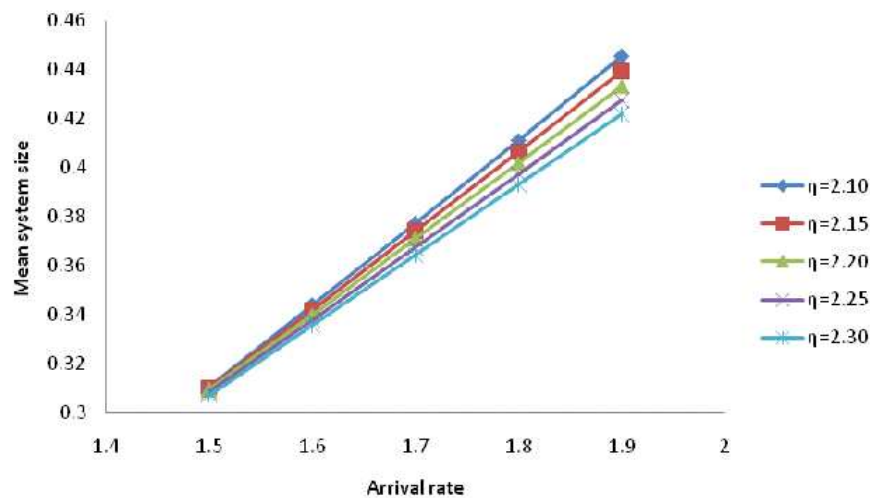
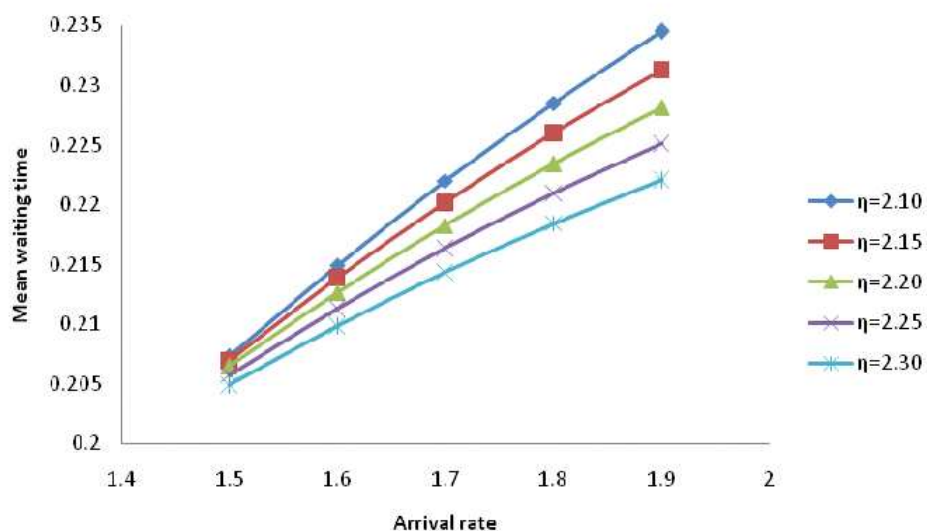
The corresponding graphs have been drawn for  $\lambda$  versus  $L_b$  and  $\lambda$  versus  $W_b$  and are shown in Figure 1 and in Figure 2, respectively. From the graphs it is seen that as  $\lambda$  increases both  $L_b$  and  $W_b$  increases for various values of  $\eta$ . Again fixing the values of  $z_1 = 0.8$ ,  $\mu_v = 5$ ,  $\mu_b = 11$ ,  $\mu_{r_1} = 2$ ,  $\mu_{r_2} = 4$ ,  $\alpha_1 = 1$ ,  $\alpha_2 = 1$  and ranging the values of  $\lambda$  from 1.5 to 1.9 in steps of 0.1 and varying the values of  $\eta$  from 2.10 to 2.30 in steps of 0.05, we calculated the corresponding values of  $L_v$  and  $W_v$  for multiple working vacation and tabulated in Table 3 and in Table 4, respectively.

Figure 1. Arrival rate ( $\lambda$ ) versus mean system size ( $L_b$ ) in regular service periodFigure 2. Arrival rate ( $\lambda$ ) versus mean waiting time ( $W_b$ ) in regular service periodTable 3. Arrival rate ( $\lambda$ ) versus mean system size ( $L_v$ ) in WV period

$\lambda \backslash \eta$	2.10	2.15	2.20	2.25	2.30
1.5	0.310922	0.310464	0.309687	0.308635	0.307349
1.6	0.343920	0.342170	0.340202	0.338051	0.335746
1.7	0.377454	0.374326	0.371091	0.367771	0.364388
1.8	0.411382	0.406801	0.402232	0.397684	0.393166
1.9	0.445566	0.439468	0.433507	0.427679	0.421979

Table 4. Arrival rate ( $\lambda$ ) versus mean waiting time ( $W_v$ ) in WV period

$\lambda \backslash \eta$	2.10	2.15	2.20	2.25	2.30
1.5	0.207281	0.206976	0.206458	0.205757	0.204899
1.6	0.214950	0.213856	0.212626	0.211282	0.209842
1.7	0.222032	0.220192	0.218289	0.216336	0.214346
1.8	0.228545	0.226000	0.223462	0.220936	0.218426
1.9	0.234509	0.231299	0.228162	0.225094	0.222094

Figure 3. Arrival rate ( $\lambda$ ) versus mean system size ( $L_v$ ) in WV periodFigure 4. Arrival rate ( $\lambda$ ) versus mean waiting time ( $W_v$ ) in WV period

The corresponding graphs have been drawn for  $\lambda$  versus  $L_v$  and  $\lambda$  versus  $W_v$  and are shown in Figure 3 and in Figure 4 respectively. From the graphs it is seen that as  $\lambda$  increases both  $L_v$  and

$W_v$  increases for various values of  $\eta$ .

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